TURBULENCE IN THE MAGNETIZED INTERSTELLAR MEDIUM

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Saturation and quenching in alpha-square dynamo

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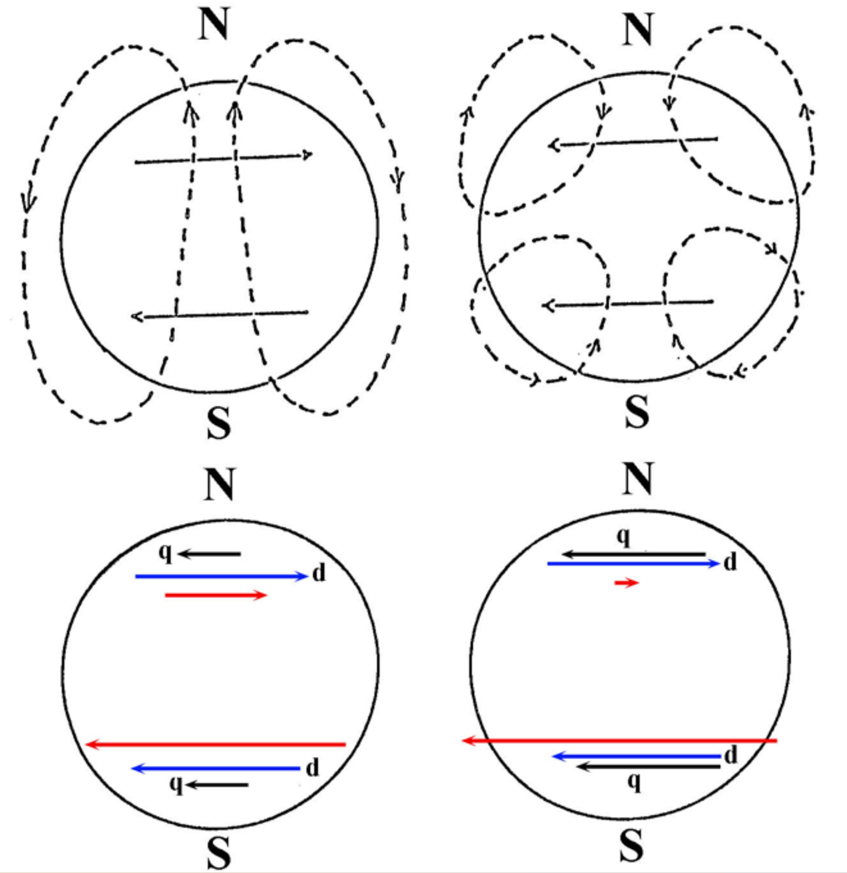
Galactic Dynamo

$$B_P \xrightarrow{\Omega} B_T$$

Differential rotation

$$B_T \xrightarrow{\alpha} B_P$$

Helicity



Magnetic helicity conservation

Growth of large-scale Component and compensation by small-scale one

Alpha-quenching

Nonlinear saturation of dynamo

Current helicity observations at Huairou

More than 1 cycle monitoring

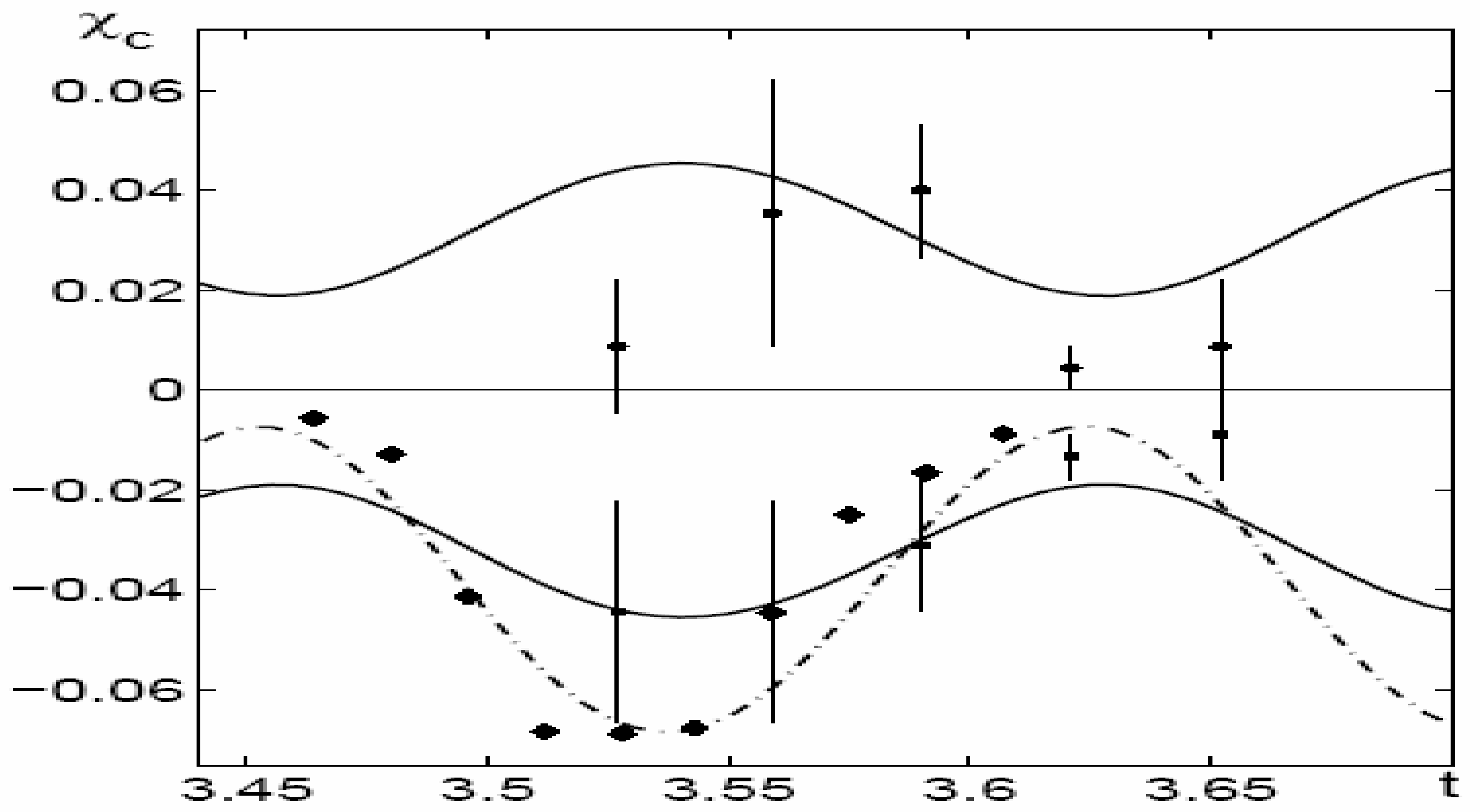


Fig. 3. The time dependence of the latitude-averaged function $\langle \chi^c \rangle$ for $D = -10^3$, $\sigma = 1$, $C = 0.01$ and $\kappa = 0.1$. The observed values of the latitude-averaged function $\langle H_c \rangle$ are shown by filled squares (for $\Theta > 0$ – Northern hemisphere, lower panel) and filled circles (for $\Theta < 0$ – Southern hemisphere, upper panel), the error-bars for $\langle H_c \rangle$ are shown by vertical lines. Dashed-dotted line indicates the time dependence of the latitude-averaged function $-6\langle B^2 \rangle$. The filled diamonds in the lower panel give the scaled averaged group sunspot numbers, R_g – see text for further details.

A Shell-Spectral Model of Galactic Dynamo

$$\partial_t B_p = i k_T \alpha B_T - k_f^2 \beta B_p,$$
$$\partial_t B_T = -i k_T \alpha B_p - k_f^2 \beta B_T.$$

Simplest model of a large-scale dynamo (alpha-square)
In principle, it could be any other spectral or grid model

$$\alpha = \alpha^u + \alpha^b.$$

Shell model

$$\left(d_t + \frac{k_n^2}{\mathrm{Re}}\right)u_n = ik_n\left\{u^*_{n+1}u^*_{n+2} - b^*_{n+1}b^*_{n+2} + \frac{1-\lambda}{\lambda^2}\left(u^*_{n-1}u^*_{n+1} - b^*_{n-1}b^*_{n+1}\right) - \frac{1}{\lambda^3}\left(u^*_{n-2}u^*_{n-1} - b^*_{n-2}b^*_{n-1}\right)\right\} + F_n,$$

$$\left(d_t + \frac{k_n^2}{\mathrm{Rm}}\right)b_n = \frac{ik_n}{\lambda(1+\lambda)}\left\{\left(u^*_{n+1}b^*_{n+2} - b^*_{n+1}u^*_{n+2}\right) + \left(u^*_{n-1}b^*_{n+1} - b^*_{n-1}u^*_{n+1}\right) + \left(u^*_{n-2}b^*_{n-1} - b^*_{n-2}u^*_{n-1}\right)\right\} + G_n.$$

Based on conservation laws of MHD + Kolomogorov description of turbulence

Conjugation

$$\alpha^u = -\frac{1}{3}\sum_n \tau_n \chi_n^u = -\frac{1}{3}\sum_n (-1)^n \tau_n k_n |u_n|^2,$$

$$\alpha^b = \frac{1}{3}\sum_n \tau_n \chi_n^j = \frac{1}{3}\sum_n (-1)^n \tau_n k_n |b_n|^2,$$

$$\beta = \frac{1}{3}\sum_n \tau_n |u_n|^2 + \beta_{\mathrm{Ohm}},$$

$$E_n^{\alpha^u} = 2ik_L\alpha_n^u (B_T B_P^* - B_P B_T^*),$$

$$E_n^{\alpha^b} = 2ik_L\alpha_n^b (B_T B_P^* - B_P B_T^*).$$

$$E_n^{\beta} = k_L^2 \beta_n E^B$$

$$F_n = f_n^I + f_n^C + f_n^B,$$

$$G_n = g_n^I + g_n^B.$$

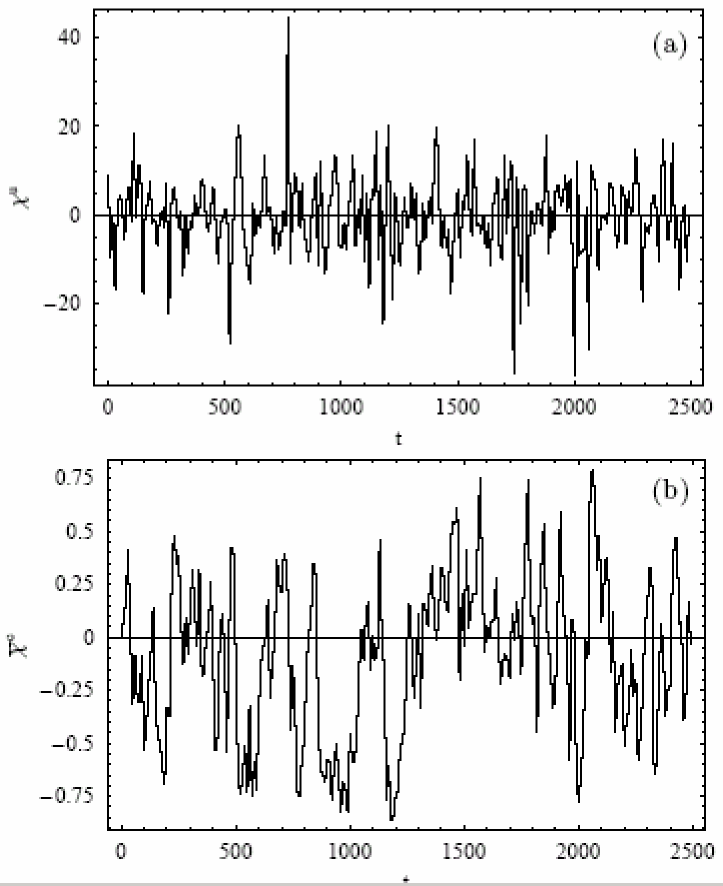
$$f_n^B = -\frac{E_n^{\alpha^u} B_n}{U_n^* B_n - U_n B_n^*},$$

$$g_n^B = -\frac{U_n Q_n}{B_n^* U_n - B_n U_n^*},$$

$$Q_n = \begin{cases} \frac{(E_n^{\alpha^b} + E_{n+1}^{\alpha^b} - E_n^{\beta} - E_{n+1}^{\beta})}{\lambda+1}, & n - \text{even} \\ \frac{(E_n^{\alpha^b} + E_{n-1}^{\alpha^b} - E_n^{\beta} - E_{n-1}^{\beta})\lambda}{\lambda+1}, & n - \text{odd} \end{cases}$$

The conditions describe the redistribution of conserved values between large- and small-scale variables

Shell model: helicities and spectra



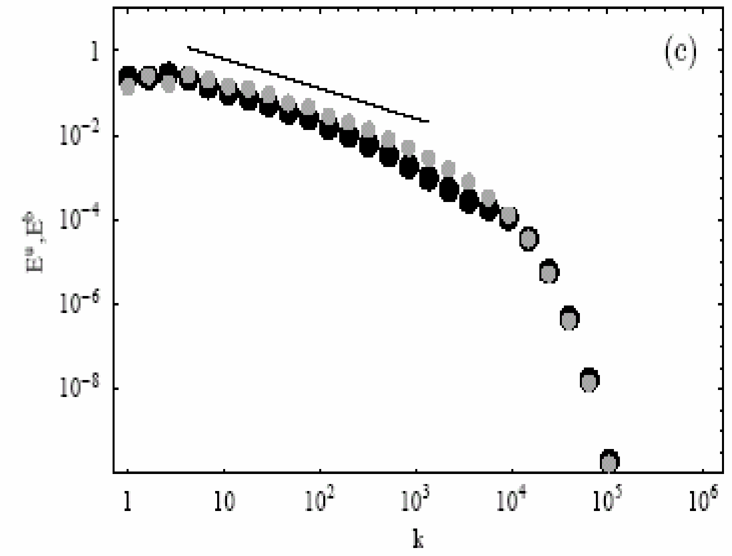


FIG. 1: Simulations of the shell model of forced MHD turbulence. Evolution of (a) hydrodynamic helicity χ^u, and (b) normalized cross-helicity $\bar{\chi}^c$. The energy spectra (c) for velocity field (black dots) and magnetic field (gray dots). The solid line shows the Kolmogorov's spectrum slope.

Shell model: Mirror asymmetry C and helicities

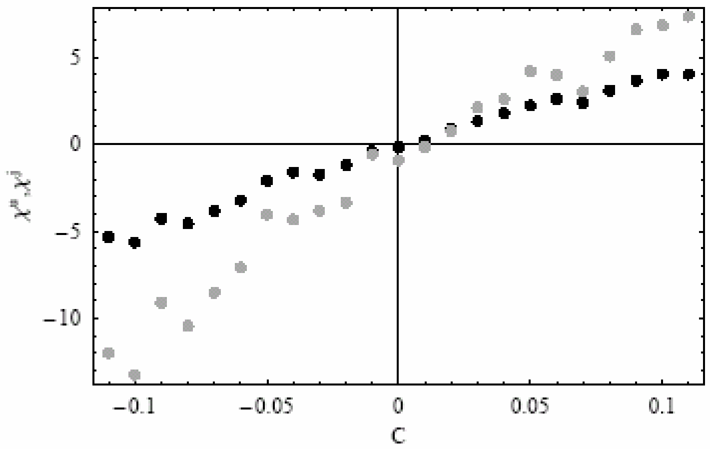


FIG. 2: Mean hydrodynamic helicity $< \chi^u >$ (black) and mean current helicity $< \chi^j >$ (gray) versus the parameter C defining the mirror asymmetry of the shell model. Any mean magnetic field is absent.

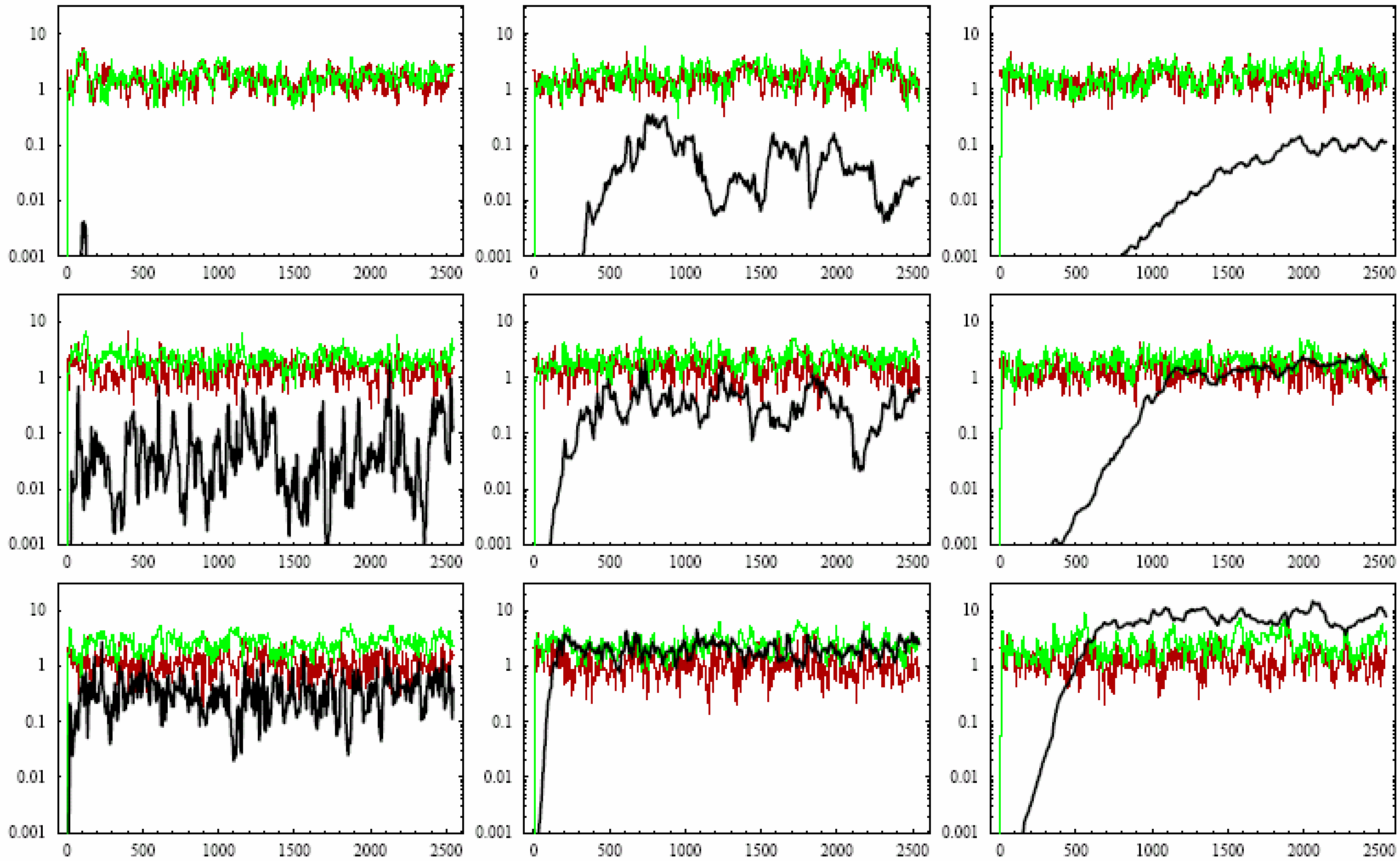


FIG. 3: (Color online) Time series for the mean magnetic field energy (thick black line), small-scale magnetic energy E^b (green line) and kinetic energy E^u (red line). Horizontal rows correspond to $C = 0, 0.04, 0.16$ (from top to bottom). Vertical columns correspond to $k_L = 1/2, 1/8, 1/32$ (from left to right). Magnetic field does not contribute to α-effect ($\alpha^B = 0$). In black and white printing out: E^b - gray, E^u - black.

$$C(\tau) = \frac{\int \widetilde{E}^B(t)\widetilde{\alpha}^u(t+\tau)dt}{(\int(\widetilde{E}^B)^2(t)dt \int(\widetilde{\alpha}^u)^2(t)dt)^{1/2}},$$

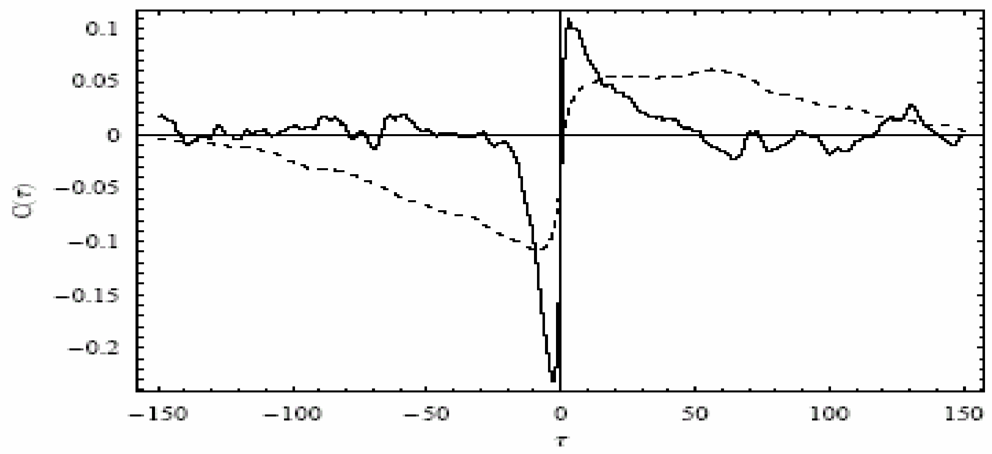


FIG. 4: Cross-correlation function of the large scale magnetic field and α^u for $C = 0.08$; $k_L = 1/2$ (solid) and $k_L = 1/16$ (dashed).

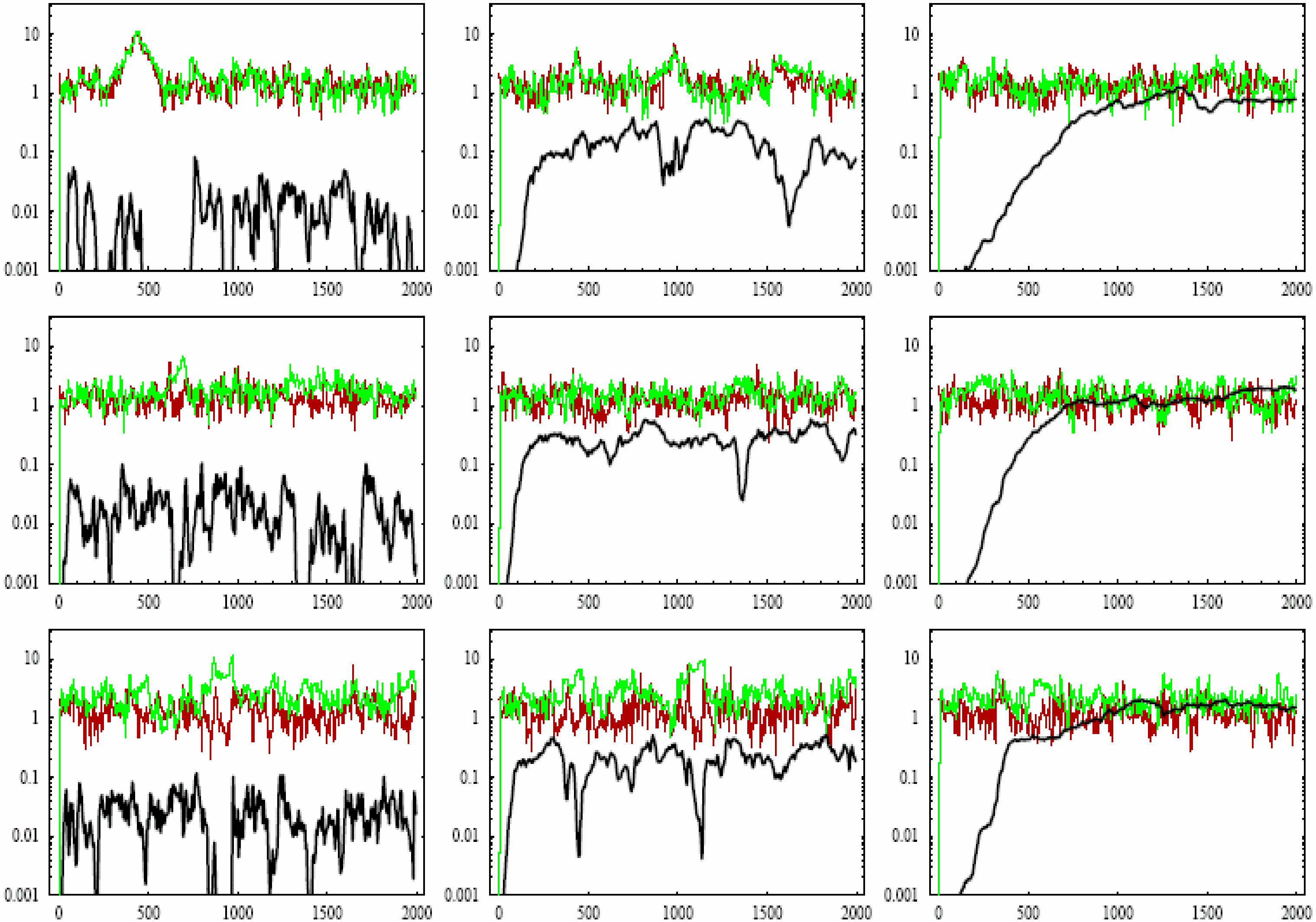


FIG. 5: (Color online) Time series for the mean magnetic field energy, small-scale magnetic energy and kinetic energy for the same parameters as in Fig. 3 but under the constraint that *only* the magnetic field contributes to the α-effect ($\alpha^u = 0$). Notations are the same as in Fig. 3.

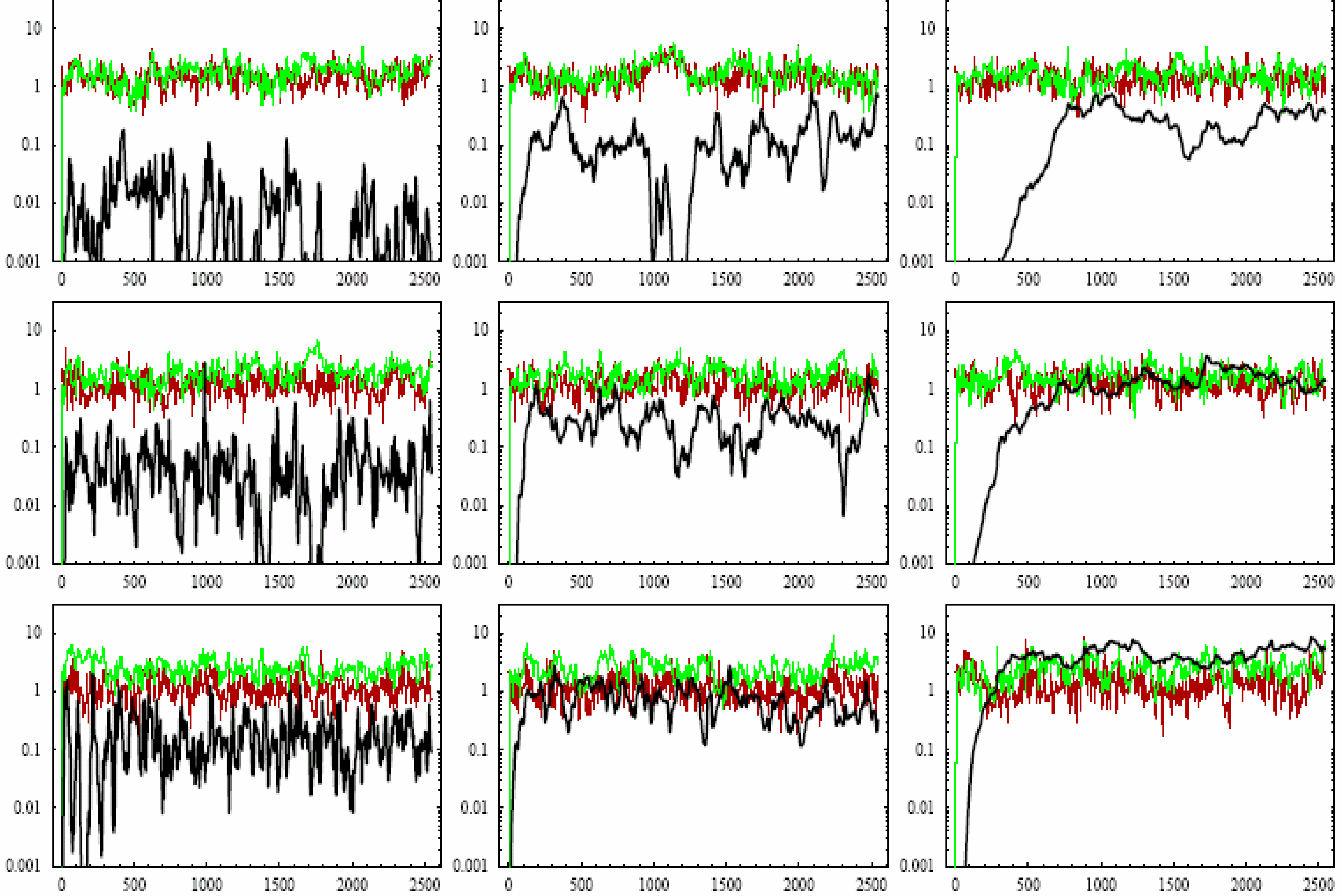


FIG. 6: (Color online) Time series for the mean magnetic field energy, small-scale magnetic energy and kinetic energy for the same parameters as in Fig. 3, but in the case of *both* contributions to the α-effect ($\alpha = \alpha^u + \alpha^b$). Notations are the same as in Fig. 3.

α^2-dynamo under low Prandtl number

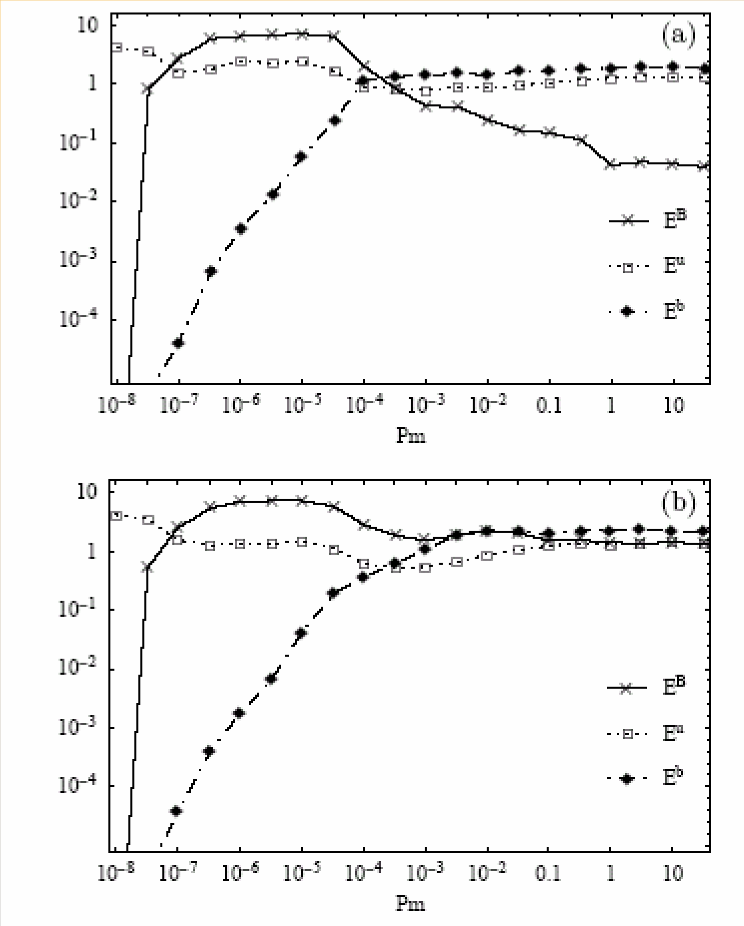


FIG. 7: Mean values of turbulent kinetic energy (empty boxes), turbulent magnetic energy (filled diamonds) and large-scale magnetic energy (crosses) versus magnetic Prandtl number. $\mathrm{Re} = 10^6$, $C = 0.08$, $k_L = 1/16$. (a) the case of *both* contributions to the α-effect ($\alpha = \alpha^u + \alpha^b$), (b) magnetic field does not contribute ($\alpha = \alpha^u$).

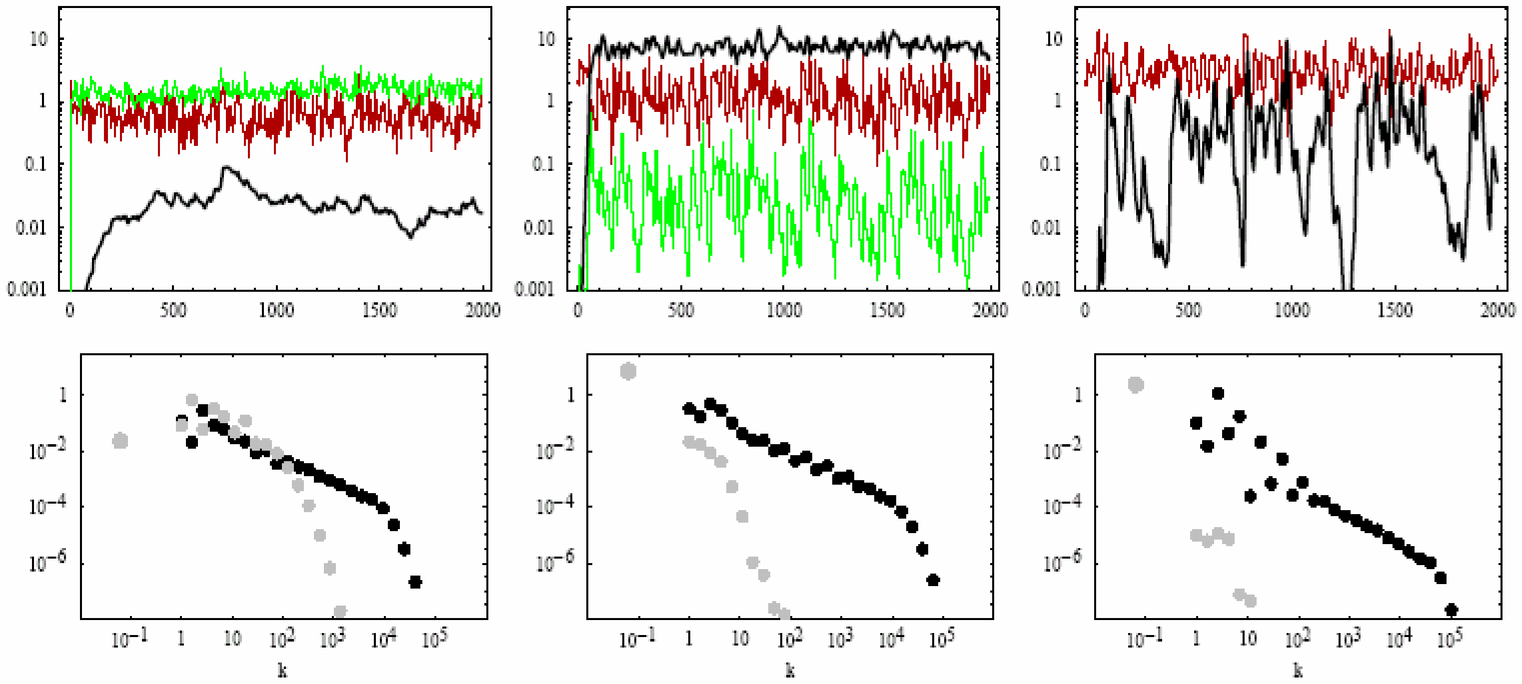


FIG. 8: (Color online) Time series (top row) and energy spectra (bottom row) of magnetic (gray dots) and kinetic (black dots) fields for some magnetic Prandtl numbers $\mathrm{Pm} = 3 \cdot 10^{-3}, 10^{-5}, 3 \cdot 10^{-8}$ (from left to right column) and $\mathrm{Re} = 10^{6}$. Value of the large-scale magnetic field energy is denoted with large gray dot.

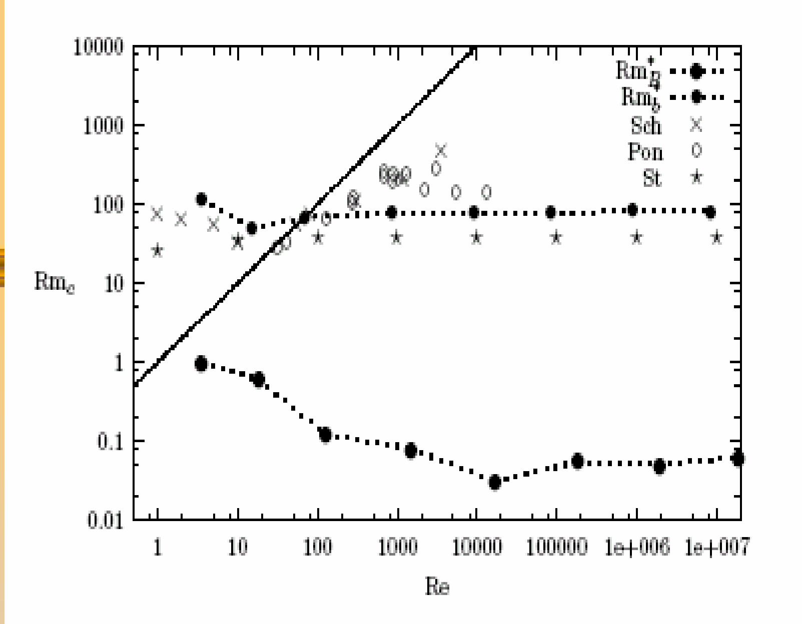


FIG. 9: Critical magnetic Reynolds number Rm_c for the large-scale α^2 dynamo (large black circles) and small-scale turbulent dynamo (small black circles) versus Reynolds number Re ($C = 0.08$, $k_L = 1/16$). Results of DNS by Schekochikhin et al. [28] are shown by croses, results of Ponty et al.[30] are shown by open circles. SM results of Stepanov and Plunian [31] are given by stars.